AP Calculus AB — Wednesday, February 19, 2025

H0218 Solutions

(#1) $f'(x) = x(x-1)$

$f' = 0 \Rightarrow x = 0$ or $x = 1$

c.p's are 0, 1

f': + | 0 | − | 1 | +

increasing on $(-\infty, 0)$ and $(1, \infty)$

decreasing on $(0,1)$

Max at $x=0$ because f' changes from + to −

Min at $x=1$ " " " " − to +

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(2) $f'(x) = (x-1)^2(x+2)^2$ cp's are $1, -2$

$f' = 0 \Rightarrow x = 1$ or $x = -2$ Sign chart: + | −2 | + | 1 | +

f is increasing on $\mathbb{R}$. There are no local extrema.

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(3) $h(x) = 2x^3 - 18x = 2x(x^2-9) = 2x(x+3)(x-3)$

Sign chart: f: − | −3 | + | 0 | − | 3 | +

negative on $(-3,0)$ and $(3,\infty)$

positive on $(-\infty,-3)$ and $(0,3)$

$h'(x) = 6x^2 - 18 = 6(x^2-3)$

local cp's at $x = \pm\sqrt{3}$

f': + | $-\sqrt{3}$ | − | $\sqrt{3}$ | +

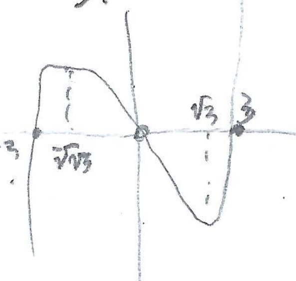
increasing on $(-\infty, -\sqrt{3})$ and $(\sqrt{3}, \infty)$

decreasing on $(-\sqrt{3}, \sqrt{3})$.

min at $x = \sqrt{3}$

max at $x = -\sqrt{3}$

no absolute extrema



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(4) $g(x) = x\sqrt{8-x^2}$ domain: $[-\sqrt{8}, \sqrt{8}]$

$$g'(x) = \sqrt{8-x^2} + x \cdot \frac{-2x}{2\sqrt{8-x^2}} = \frac{2(8-x^2) - 2x^2}{2\sqrt{8-x^2}} = \frac{16-4x^2}{2\sqrt{8-x^2}}$$

$g' = 0 \Rightarrow 16 - 4x^2 = 0 \Rightarrow 4 = x^2 \Rightarrow x = \pm 2$

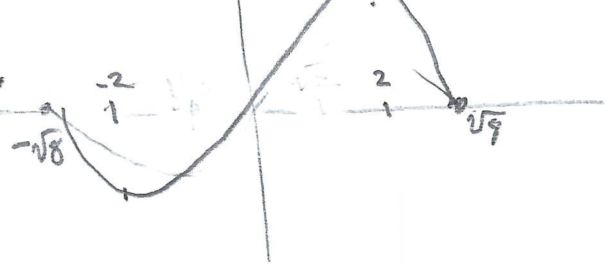
g': − | −2 | + | 2 | −

increasing on $(-2, 2)$

decreasing on $(-\sqrt{8}, -2)$ and $(2, \sqrt{8})$

$g(2) = 2\sqrt{4}$
$= 4$

Both local extrema are absolute.



H0218 cont

(5) $f(x) = ax^2 + bx + c$.

$f'(x) = 2ax + b = 0 \Rightarrow x = -\frac{b}{2a}$.

sign chart f'

$a > 0$: $-$ | $+$, at $-\frac{b}{2a}$

$a < 0$: $+$ | $-$

Increasing on $(-\frac{b}{2a}, \infty)$
decreasing on $(-\infty, -\frac{b}{2a})$
in $a>0$. Reversed if $a<0$.

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(6) $f(x) = \dfrac{x^2+1}{x}$

$g(b)$ = average rate of change of f on $[1, b]$

$$= \frac{f(b) - f(1)}{b-1} = \frac{\frac{b^2+1}{b} - \frac{2}{1}}{b-1} = \frac{b^2 - 2b + 1}{(b-1)b} = \frac{(b-1)^2}{(b-1)b} = \frac{b-1}{b} = 1 - \frac{1}{b}$$

So, $\displaystyle\lim_{b\to\infty} \frac{f(b)-f(1)}{b-1} = \lim_{b\to\infty} 1 - \frac{1}{b} = 1.$

That is, f is asymptotic to a line of slope 1,

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(7) $\int x \tan^2(x^2)\, dx = \int x(\sec^2(x^2) - 1)\, dx = \frac{1}{2}\tan(x^2) - x + C$

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(8) $\int x \cos(x)\sqrt{\sin(x^2)}\, dx = \underline{\text{oops}}$ | should be $\int x\cos(x^2)\sqrt{\sin(x^2)}\, dx$

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(9) $f(x) = x^3 - 6x^2 + 9x = x(x^2 - 6x + 9) = x(x-3)^2$

zeros: $x = 0, 3$

$f'(x) = 3x^2 - 12x + 9 = 3(x^2 - 4x + 3) = 3(x-1)(x-3)$

zeros: $x = 1, 3$

$f(1) = 1 - 6 + 9 = 4$
$f(3) = 0$

min at $x = 3$, max at $x = 1$

(10b)

- MVT
- IVT
- EVT